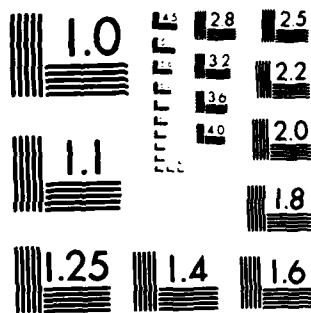AD-A144 387   ADAPTIVE CONTROL WITH VARIABLE DEAD-ZONE NONLINEARITIES   1/1
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ADAPTIVE CONTROL WITH VARIABLE DEAD-ZONE NONLINEARITIES*

by

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Abstract

It has been found that fixed error dead-zones
s defined in the existing literature result in
erious degradation of performance, due to the con-
ervativeness which characterizes the determination
f their width. In the present paper, variable
width dead-zones are derived for the adaptive con-
trol of plants with unmodeled dynamics. The der-
ivation makes use of information available about
the unmodeled dynamics both a priori as well as
during the adaptation process, so as to stabilize
the adaptive loop and at the same time overcome the
conservativeness and performance limitations of
fixed-dead zone adaptive or fixed gain controllers.

1. INTRODUCTION

Research in recent years has shown that adap-
tive control algorithms which, under ideal assump-
tions, have been proven globally asymptotically
stable, indeed exhibit unstable behavior in cir-
cumstances under which those assumptions are even
slightly violated. Of the two instability mech-
anisms identified for these algorithms, -commonly
referred to as "gain" and "phase" instability mech-
anisms [1],- the former is more unavoidable and is
triggered by the controller parameter
drift which occurs as a result of nonzero output
errors. These are a consequence of the fact that,
in the presence of unmodeled dynamics and/or
(persistent) disturbances there can be no perfect
(transfer function) matching between the compensated
plant and the reference model over all frequencies,
even if "sufficiency of excitation" for the "nom-
inal" model order is guaranteed.

Perfect matching, on the other hand, translates
into zero output (tracking) error, under ideal as-
sumptions, and has been the basis for the parameter
adjustment laws; only when the output error is zero
does adaptation stop. Clearly, then, by design,
any nonzero output error is instantaneously attrib-
uted to parameter errors. Furthermore, there is
nothing in the mathematics of the adjustment mech-
anisms, as they currently stand, to prevent gain
drift due to error sources other than parameters,
as for example happens even in cases of "exact
modeling", with "sufficiency of excitation", where
convergence to the "desired" parameter values has
been achieved momentarily; extraneous
disturbances entering at that point can cause the
parameters to drift from their "desired" values.

Consequently then, the zero tracking error
requirement must be suitably relaxed. The ration-
ale is that no parameter adjustment should take
place when the output error(s) are due to disturb-
ances and/or unmodeled dynamics. This can be
achieved on an existing algorithm by a dead-zone
nonlinearity, in the parameter adjustment law,
whose width depends on the contribution of the
disturbances and/or unmodeled dynamics to the out-
put error.

The idea of a dead-zone nonlinearity in the
parameter update law to avoid the effect of disturb-
ances on adaptation was first introduced for in-
direct adaptive algorithms by Egardt in 1980 [2]
and was later amplified by Samson [3]. Also, in
1982 Peterson and Narendra used a dead-zone non-
linearity to prove stability for a class of direct
algorithms in the presence of bounded disturbances
with no unmodeled dynamics [4]. However, the width
of the dead zone was chosen to be constant and had
to be based on a very conservative bound so that it
yielded only marginally stable systems with ex-
tremely poor model tracking as the examples in [4]
seem to suggest.

Consequently, obtaining non-fixed accurate
bounds for the disturbance and high-frequency
dynamics contributions to the output error is cru-
cial to overcoming the conservativeness of the
dead-zone width which they define. This depends on
the ability to translate frequency domain
magnitude bounds, most naturally expressed by
ℓ^2-norms into time-domain magnitude bounds
of instantaneously measured quantities, most nat-
urally expressed by ℓ^∞- or, for our purposes, ℓ^∞-
norms which are much less conservative than ℓ^∞ -
norms.

This paper discusses the use of a deadzone,
whose width is adjustable on line, to adaptively
control a plant with unmodeled dynamics, with the
objective of maintaining its stability and minimi-
zing the adverse effects of a conservative dead-
zone width to its performance. Due to space consid-
erations we do not treat the case of output disturb-
ances here; also, the topic of disturbances addi-
tionally includes a fixed disturbances rejection
mechanism that introduces a modification in the bas-
ic structure of the MRAC system so is to merit
separate attention.

Section 2 of this paper contains a generic norm
translation problem and develops a set of tools

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required for its solution. Section 3 applies the results of the previous section to the familiar MLV algorithm of the Model Reference type. Other algorithms can be treated similarly. Section 4 discusses the stability of the variable width dead-zone adaptive system and, finally, Section 5 contains the concluding remarks.

2. MATHEMATICAL PRELIMINARIES

In this section we develop the necessary tools for the definition of the variable width dead zone. As was already pointed out in the introduction, the objective is to find satisfactory bounds for the contribution of the unmodeled dynamics to the output of the adaptively controlled process, so that an "accurate" error dead-zone can be defined.

The process, complete with unmodeled dynamics is assumed to be of the Doyle-Stein type, with the high frequency dynamics entering multiplicatively; i.e.

$$g(s) = g_p(s)(1+\ell(s))$$

where $g(s)$ is the actual plant transfer function, $g_p(s)$ its modeled part and $\ell(s)$ the unmodeled dynamics. Typically, a bound on the magnitude of $\ell(s)$ is assumed to be negligible for frequencies below crossover, becoming only appreciable for higher frequencies; no phase information can be assumed. The problem is then to find a bound for the output of the adaptively controlled process due to $\ell(s)$. This is achieved in two stages, as the following subsections indicate. We further remark here that, due to feedback in the adaptive loop, the unmodeled dynamics indirectly influence all the state variables of the nominal adaptive loop, with the magnitude of their contribution depending on the nature of the inputs to the plant.

2.1 Transfer Function Magnitude Bounding

In this subsection we will derive a bound $\phi(\omega)$ on the magnitude of the frequency response of a special class of transfer functions, that typically arise in MRAC systems. Consider an LTI transfer function $G(\underline{\theta},s)$ of the form

$$G(\underline{\theta},s) = M(s) \left\{ \frac{\ell(s)}{1+A(\underline{\theta},s)[1+\ell(s)]} \right\} \tag{2a}$$

where

$M(s)$ is a completely known stable LTI transfer function

$$1+A(\underline{\theta},s) = \prod_{i=1}^{n} (s+p_i(\underline{\theta})) \tag{2b}$$

$$\underline{\theta}_\ell \le \underline{\theta} \le \underline{\theta}_u$$

$\underline{\theta}$ unknown constant parameter vector with specified bounds (2c)

$$\underline{p}_i(\underline{\theta}) \le 0 \qquad i=1,\dots,n \tag{2d}$$

$$||\ell(j\omega)|| \le \ell_o(\omega) \quad \text{for known } \ell_o(\omega) > 0 \tag{2e}$$

Note that when $\ell(s)=0$, $G(\underline{\theta},s)=M(s)$, and therefore stable. In the context of MRAS, $G(\underline{\theta},s)$ represents

the actual transfer function of a plant with feedback, designed to follow a reference model which is prescribed by the transfer function $M(s)$. In the absence of unmodeled dynamics, $\ell(s)$, perfect matching is possible.

When $\ell(s)\neq0$, the stability of $G(\underline{\theta},s)$ can be ensured by (requiring) enforcing the condition

$$1+A(\underline{\theta},j\omega)[1+\ell(j\omega)]\neq0 \tag{3}$$

This condition is satisfied if, for all $\underline{\theta}$ in the space of admissible parameters the following is true:

$$||\ell(j\omega)|| \le \ell_o(\omega) \le 1+||A^{-1}(\underline{\theta},j\omega)|| \tag{4}$$

Assuming this condition is true, we proceed to derive an upper bound on $||\overline{G(\underline{\theta},j\omega)}||$, parametrized by $||M(j\omega)||$. From eqn. (2),

$$G(\underline{\theta},j\omega)=M(j\omega) \left\{ \frac{\ell(j\omega)}{D(\underline{\theta},j\omega)+\ell(j\omega)A(\underline{\theta},j\omega)} \right\} \tag{5}$$

$$\text{where } D(\underline{\theta},j\omega)\equiv1+A(\underline{\theta},j\omega)$$

Next, representing $\ell(j\omega)$, $D(\underline{\theta},j\omega)$ and $A(\underline{\theta},j\omega)$ in polar form,

$$||G(\underline{\theta},j\omega)||\le||M(j\omega)|| \left\| \frac{||\ell(j\omega)||e^{j\phi_\ell}}{||D(\underline{\theta},j\omega)||e^{j\phi_D(\underline{\theta})}+||\ell(j\omega)A(\underline{\theta},j\omega)||e^{j(\phi_A(\underline{\theta})+\phi_\ell)}} \right\| \tag{6}$$

An upper bound for $||G(\underline{\theta},j\omega)||$ can be found by maximizing and minimizing respectively the values of the numerator and denominator terms in (6). The denominator achieves its smallest value if the vectors $\ell(j\omega)A(\underline{\theta},j\omega)$ and $D(\underline{\theta},j\omega)$ are oppositely aligned and, in addition, $||\ell(j\omega)||$ achieves its maximum allowable value for the θ-interval of interest. The above two conditions are satisfied if the phase angles are such that $\phi_\ell+\phi_A(\underline{\theta})=\phi_D(\underline{\theta})+\pi$ and $||\ell(j\omega)||=\ell_o(\omega)$. Note that condition (4) for the stability of $G(\underline{\theta},s)$ ensures that $||\ell_o(j\omega)A(\underline{\theta},j\omega)||<||D(\underline{\theta},j\omega)||$ and, indeed, the choice $||\ell(j\omega)||=\ell_o(\omega)$ guarantees minimization of the denominator of $G(\underline{\theta},j\omega)$ with stability maintained.

Unlike the denominator, the numerator magnitude is independent of ϕ_ℓ, and is directly maximized by choosing $||\ell(j\omega)||=\ell_o(\omega)$ and ϕ_ℓ appropriately to satisfy the denominator phase angle condition. Consequently, from eqn. (6) we can now write an upper bound for $G(\underline{\theta},j\omega)$ as follows:

$$||\overline{G(\underline{\theta},j\omega)}||=||M(j\omega)|| \frac{\ell_o(\omega)}{||D(\underline{\theta},j\omega)||-\ell_o(\omega)||A(\underline{\theta},j\omega)||} \tag{7}$$

Finally, we may search the space of allowable values of $\underline{\theta}$ to determine the desired transfer function bound

$$\phi(\omega) \triangleq \max_{\underline{\theta}} ||\overline{G(\underline{\theta},j\omega)}|| \tag{8}$$

and eqns. (25) and (26), it is straightforward to conclude that

$$\dot{\tilde{k}}(t) \in L^2 \qquad (27)$$

From this point on, the proof uses standard arguments, for the boundedness of u, w, ζ, as they first appeared in [7] and outlined in [4]. We only remark here that, in our case, $|v(t)| < |\varepsilon(t)|$ and, furthermore, $v(t) = 0[\sup_{\tau > T} ||w(\tau)||]$, as follows from eqn. (22) and fig. 4. The reader is again referred to [6] for all the details of the stability arguments.
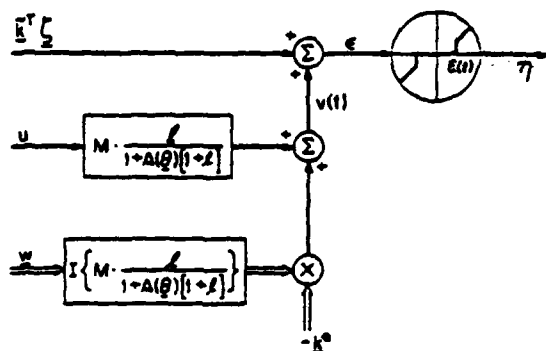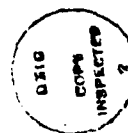

Figure 5:

6. CONCLUSIONS

A variable dead-zone nonlinearity was introduced in a standard model reference adaptive control algorithm to maintain its stability in the presence of unmodeled dynamics. The variable width dead-zone is determined on-line on the basis of prior information about plant parameter bounds and unmodeled dynamics as well as about information obtained during adaptation. Besides maintaining stability, the algorithm is able to overcome the conservativeness of fixed dead-zone on exponential forgetting factor adaptation mechanisms [4],[8], as simulation results show. Due to space limitations, those are deferred until the conference presentation of the paper.

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In the following subsection we will use $\phi(\omega)$ in order to bound the output of $G(\underline{\theta}, j\omega)$, in an absolute value sense, given an input $x(t)$.

2.2 Absolute Value Output Bounding

Consider the system shown in figure 1 with input $x(t)$, output $y(t)$ and $G(\underline{\theta}, s)$ as defined before
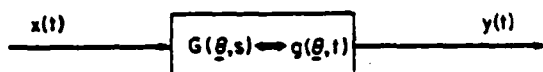
x(t) ──→ │ $G(\underline{\theta}, s) \Longleftrightarrow g(\underline{\theta}, t)$ │ ──→ y(t)

Figure 1:

By taking the inverse Laplace transfer of $G(\underline{\theta}, s)$, we obtain

$$L^{-1}\{\underline{G}(\underline{\theta}, s)\} = g(\underline{\theta}, t) \qquad (9)$$

Furthermore, since $g(\underline{\theta}, t)$ represents a linear-time-invariant system, by definition of $G(\underline{\theta}, s)$, the output $y(t)$ is given by

$$y(t) = g(\underline{\theta}, t) * x(t) =$$
$$= \int_{-\infty}^{t} d\tau \, g(\underline{\theta}, t-\tau) x(\tau) \qquad (10)$$

Substituting now in (10) for $g(\underline{\theta}, t-\tau)$ the expression for the inverse Laplace transform of $G(\underline{\theta}, s)$ and recalling, further, that $G(\underline{\theta}, s)$ is a stable transfer function, we can write

$$y(t) = \int_{-\infty}^{t} d\tau \, \frac{1}{2\pi j} \int_{-j\infty}^{j\infty} ds \, G(\underline{\theta}, s) e^{s(t-\tau)} \, x(\tau) =$$
$$= \frac{1}{2\pi} \int_{-\infty}^{\infty} d\omega \, G(\underline{\theta}, j\omega) e^{j\omega t} \int_{-\infty}^{t} d\tau \, x(\tau) e^{-j\omega\tau} \qquad (11)$$

Next, with $u(t)$ representing the unit step function we have

$$F\{x(t)u(-t)\} = \int_{-\infty}^{\infty} d\tau \, x(\tau) u(-\tau) e^{-j\omega\tau} = \int_{-\infty}^{t} d\tau \, x(\tau) e^{-j\omega\tau} \qquad (12)$$

Define $F\{x(t)u(-\tau)\}$ with the symbol $\overline{X}(j\omega)$ and substitute in (11). Then

$$y(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} d\omega \, G(\underline{\theta}, j\omega) \overline{X}(j\omega) e^{j\omega t} \qquad (13)$$

By the Cauchy-Schwartz inequality it follows from (13) that

$$|y(t)| \leq \frac{1}{2\pi} \int_{-\infty}^{\infty} d\omega \, ||G(\underline{\theta}, j\omega)|| \, ||\overline{X}(j\omega)|| \qquad (14)$$

But from the previous subsection $\phi(\omega)$ was determined such that

$$\phi(\omega) \geq ||G(\underline{\theta}, j\omega)|| \qquad (15)$$

Hence, (14) becomes,

$$|y(t)| \leq \frac{1}{2\pi} \int_{-\infty}^{\infty} d\omega \, \phi(\omega) ||\overline{X}(j\omega)|| \equiv \overline{y}(t) \qquad (16)$$

which admittedly represents a looser bound on the absolute value of $y(t)$ than eqn. (14). However, the bound $\overline{y}(t)$ can be calculated more readily. The inequality (16) has the following interpretation. Given a bound on the frequency response of a system, a bound on the magnitude of its output due to an input $x(t)$ can be calculated at any instant of time by using the time history of the system input up to and including that instant of time. We note here again the time dependence of $\overline{X}(j\omega)$ according to eqn. (12).

3. MRAC WITH RELAXED TRACKING ERROR CRITERION

In this section we employ the results of section 2 to derive a variable dead-zone width for the parameter update of the N-L-V algorithm, which overcomes the conservativeness of the fixed width Peterson-Narendra scheme [4]. Before we proceed, we briefly review the concept of parameter update using a dead-zone nonlinearity.

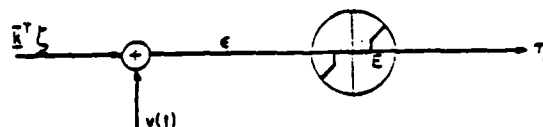In [4] the authors have shown stability of the system depicted in figure 2 below.

Figure 2:

In fig. 2 the standard notation is used, with $\underline{k}$ representing parameter errors, $\underline{z}$ filtered auxiliary) state variables $v(t)$ output deterministic disturbances of bounded magnitude, differentiable and uniformly continuous, ϵ the output error with disturbances; η represents the (part of the) error actually used in the parameter adaptive law and is obtained by ϵ passed through the dead-zone of width ξ.

It is not our purpose here to present the details of how the above error system as shown in fig. 2, is arrived at. The reader is instead referred to [4] for those as well as the stability proof of that modified algorithm. We simply present here the parameter adaptive laws, with the dead-zone nonlinearity in their simplest form, for the sake of completing the problem description which forms the basis for the developments in the present

paper. The parameter adjustment is as described by eqns. (17) below.

$$\dot{\underline{k}}(t) = \dot{\tilde{k}}(t) = \frac{-\zeta(t)\eta(t)}{1+\alpha\zeta^T(t)\zeta(t)} \qquad \alpha > 0 \qquad (17a)$$

$$\text{with} \quad \eta(t) = \begin{cases} e(t) \cdot & |e(t)| > \varepsilon \\ 0 & |e(t)| < \varepsilon \end{cases} \qquad (17b)$$

where ε represents the width of the dead-zero.

Although in [4] the discussion is not particularly enlightening as to how exactly the magnitude ε is decided upon, it is the present authors' conclusion that most likely, in [4]

$$\varepsilon \geq ||v(t)||_\infty = \max_t |v(t)| \qquad (18)$$

and, therefore, is very conservative as the same authors have pointed out in [5]. We next proceed to analyze the original NLV Model Reference algorithm, as represented in Fig. 3, with the plant dynamics now replaced by the actual plant $\underline{P}(\underline{\theta},s)[1+\ell(s)]$.
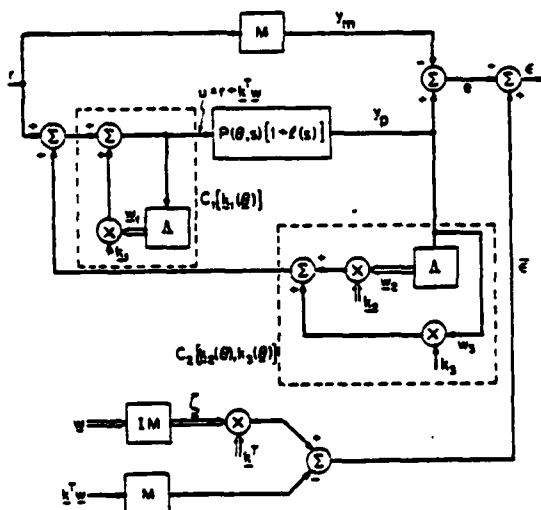

Figure 3:

The component $\underline{P}(\underline{\theta},s)$ incorporates the designer's knowledge of the dominant, low frequency response of the plant, including a vector $\underline{\theta}$ of uncertain parameters, known only within precomputable bounds. The standard MRAC assumptions about the plant hold

for $P(\underline{\theta},s)$. That is, the designer knows
(i)   an upper bound on the relative degree n^* of $P(\underline{\theta},s)$
(ii) an upper bound on the degree n of $P(\underline{\theta},s)$
(iii) that $P(\underline{\theta},s)$ is minimum phase
(iv) the sign of the high frequency gain of $P(\underline{\theta},s)$

The $\ell(s)$ part of the plant represents the (multiplicative) uncertainty associated with the nominal plant $P(\underline{\theta},s)$. This uncertainty is due to high frequency dynamics, which are assumed of unspecified structure but satisfy a magnitude constraint $||\ell(j\omega)|| \leq \ell_o(\omega)$, as already mentioned. Further, we note that the "actual" plant representation $\underline{P}(\underline{\theta},s)[1+\ell(s)]$ will not in general satisfy any of the four standard assumptions listed above. This fact becomes pivotal in the inability of the adaptive controller to achieve transfer function matching between the compensated plant and the reference model. As a result, it becomes impossible for general inputs to drive the tracking (output) error to zero. However, one may expect the tracking error to be small, if the plant is excited by signals with dominant low frequency content over the range where $P(\underline{\theta},s)$ is a good approximation to the actual plant transfer function.

In what follows we will next show that the error system underlying the structure in figure 3 differs from that of the same structure, as shown in fig. 2, where $\ell(s)=0$, only by an additive perturbation term in the output. This term can be bounded using the results of Section 2 and a variable width dead-zone can be defined for the adaptation mechanism. Stability of the scheme is subsequently discussed in Section 5.

We start by considering first the case where $\ell(s)=0$ in Figure 3. In this case the standard MRAC assumptions about the plant are true. There exists a vector $\underline{k}^*(\underline{\theta})$ of fixed gains which, when applied to the system, results in matching of the compensated plant transfer function with that of the model. We adopt the shorthand notation $C_1(\underline{\theta})$ and $C_2(\underline{\theta})$ to indicate the LTI transfer functions $C_1(\underline{k}_1^*(\underline{\theta}))$ and $C_2(\underline{k}_2^*(\underline{\theta}),k_3^*(\underline{\theta}))$ respectively. Now assuming $\ell(s)\neq 0$ but maintaining the same definitions of $C_1(k_1^*(\underline{\theta})) = C_1(\underline{\theta})$ and $C_2(k_2^*(\theta),k_3^*(\theta))$, = $C_2(\underline{\theta})$ based on the reduced model, we may derive an expression for the error system as follows, where for convenience, the argument s has been suppressed throughout.

$$\frac{Y_p}{R} = \frac{C_1(\underline{\theta})P(\underline{\theta})[1+\ell]}{1+C_1(\underline{\theta})C_2(\underline{\theta})P(\underline{\theta})[1+\ell]}$$

$$= \frac{C_1(\underline{\theta})P(\underline{\theta})}{1+C_1(\underline{\theta})C_2(\underline{\theta})P(\underline{\theta})} + \frac{C_1(\underline{\theta})P(\theta)}{1+C_1(\underline{\theta})C_2(\underline{\theta})P(\underline{\theta})} \cdot$$

$$\frac{\ell}{1+C_1(\underline{\theta})C_2(\underline{\theta})P(\theta)[1+\ell]} \qquad (19)$$

By definition of $C_1(\underline{\theta})$ and $C_2(\underline{\theta})$ we have

$$\frac{C_1(\underline{\theta})P(\underline{\theta})}{1+C_1(\underline{\theta})C_2(\underline{\theta})P(\underline{\theta})} = M \qquad (20)$$

Using this fact and introducing the notation $A(\underline{\theta}) \equiv C_1(\theta)C_2(\underline{\theta})P(\underline{\theta})$ we can write eqn. (19) in a more compact form as

$$\frac{y_p}{R} = M+M \cdot \frac{\ell}{1+A(\underline{\theta})[1+\ell]} \quad (21)$$

Next, defining $\underline{\tilde{k}}(t) = \underline{k}(t)-\underline{k}^*$, referring to fig. 3 and interchanging time domain and transformed quantities, we derive an expression for ϵ as given in eqn. (22).

$$\epsilon = \bar{\epsilon} + \frac{y_p}{R} \cdot R + \frac{y_p}{R} \cdot \tilde{k}^T\underline{w} - \frac{y_m}{R} \cdot R$$

$$= \underline{\tilde{k}}^T\underline{\zeta} + M \cdot \frac{\ell}{1+A(\underline{\theta})[1+\ell]} \cdot (\underline{\tilde{k}}^T\underline{w}+r) \quad (22)$$

For the case where $\ell(s)=0$, this result reduces to the standard augmented MRAC error system of Narendra, Lin and Valavani with $L^{-1} = M$. The new error system is shown in Figure (4) with a variable dead zone non-linearity added to the output signal path.
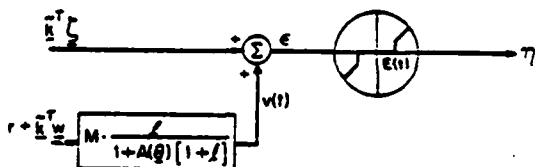

Figure 4:

We observe that the system is of the form shown in fig. 2. In order to specify a stable adaptive law, we need to find a bounding signal $E(t) \geq |v(t)| \forall t$.

We may redraw the error system using the fact that $\underline{\tilde{k}}=\underline{k}^*-\underline{k}$ and the input to the plant in Figure 5 is $u=\underline{k}^T\underline{w}+r$. The resulting representation is shown in Figure 5 below.

From precomputed bounds on $\underline{\theta}$, bounds on k^* can be precomputed also. We now make the definition

$$\hat{\underline{k}}^* = |\max_{\underline{\theta}} \underline{k}^*| \quad (23)$$

where the maximization over $\underline{\theta}$ is carried out individually over every component of $\underline{k}^*$. Using (23) in conjuction with the results of Section 2 it readily follows that an upper bound $E(t)$ for $v(t)$ can be computed. More specifically, we can write

$$E(t) \equiv \int_{-\infty}^{\infty} \phi(\omega) \left\{ |\bar{U}(j\omega)| + \sum_{i=1}^{2n-1} |\bar{W}_i(j\omega)||\hat{k}_i^*| \right\} d\omega \quad (24)$$

where eqn. (12) has been used for calculation of the transforms $\bar{U}(j\omega)$ and $\bar{W}_i(j\omega)$ for the input $u(t)$ and signals $\omega_i(t)$ respectively. An adaptation law of the form described in eqn. (17) can then be employed with the width of the dead zone defined by eqn. (24). The resulting scheme is stable and an outline of its stability proof is given in the following section.

5. STABILITY

The stability proof of the proposed algorithm with variable dead-zone follows along very similar lines for the most part with that in [4]. However, in the present case it is additionally conditioned on the reference model definition and the admissible parameter set, as eqn. (4) of section 2.1 implies. More specifically, the space of admissible parameters is implicitly defined through the reference model by eqn. (4), in conjunction with condition (2c) and is such that the desired (class of) reference model(s) remains stable in the presence of the unmodeled dynamics $\ell(s)$ of the plant. This is a standard and reasonable assumption made in the design of all fixed parameter controllers as well. Due to space considerations we will not elaborate on this further but will instead refer the reader to [6] for more details.

Consequently, given eqn. (4), which is fundamental even for a non-adaptive design, the effect of unmodeled dynamics can be represented as an output perturbation $v(t)$ as suggested in eqn. (22) and depicted in fig. 4. $v(t)$ is the output of a stable linear system which is bounded for bounded inputs. We next proceed to outline the steps for proving boundedness of $\tilde{k},u,\zeta,w$ and the output error ϵ.

The boundedness of $\tilde{k}$ follows directly from the standard Lyapunov function definition

$(V(\underline{\tilde{k}}) = \frac{1}{2} (\underline{\tilde{k}}^T\underline{\tilde{k}})$ and the adaptation law (17a) in conjunction with eqn. (22) where $E(t)$ is defined. Also, from the definition of the dead-zone, the term $\underline{\tilde{k}}^T\underline{\dot{k}}$ can be bounded above and below by bounds of the form

$$f_1[E(t)]|\underline{\tilde{k}}^T\underline{\zeta}+v(t)| \leq |\underline{\tilde{k}}^T\underline{\dot{k}}| \leq f_2[E(t)] \cdot |\underline{\tilde{k}}^T\underline{\zeta}+v(t)| \quad (25)$$

where $f_1(\cdot)$ and $f_2(\cdot)$ are continuous functions of $E(t)$.

Next, by the definition of the Lyapunov function, its time derivative, in conjunction with the adaptation law given by eqn. (12a), can be written as

$$\dot{V} = \underline{\tilde{k}}^T\underline{\dot{k}} = -\frac{\tilde{k}^T\underline{\zeta}\eta}{1+\underline{\zeta}^T\underline{\zeta}} \quad (26)$$

From the fact that

$$\int_0^\infty \dot{V}(\tau)d\tau < \infty$$